Shells, Anti-Shells and Modes in Nuclear Fission

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Shells and Anti-Shells

Nuclear Masses in the Liquid Drop Model (LDM):

\[ M(A,Z) = a_V A + a_S A^{\frac{2}{3}} + a_C Z^2 / A^{\frac{2}{3}} + a_I (N-Z)^2 / A - \delta(A) \]

Volume, Surface, Coulomb, Symmetry, Pairing

“SHELL” corrections:

Macroscopic LDM describes average masses. Microscopic nuclear structure necessitates corrections.

\[ \delta W = M_{\text{exp}} - M_{\text{LDM}} \]

Myers-Swiatecki 1966: neutron and proton ranges with \( \delta W < 0 \) and \( \delta W > 0 \) alternate

\[ \delta W < 0: \text{ nuclei are more tightly bound than in LDM: “SHELLS”} \]

\[ \delta W > 0: \text{ nuclei are less tightly bound than in LDM: “ANTI-SHELLS”} \]

Shell corrections for actual fission fragments.

Shell for \( A \approx 130 \text{ u} \)
Anti-shell for \( A \approx 110 - 120 \text{ u} \) and \( A > 150 \text{ u} \)
Properties
of shell and anti-shell nuclei

Stiffness of nuclei

\[ E_{\text{Deformation}} = \alpha (D - R_0)^2 \]
\[ \alpha = \text{stiffness} \]
\[ D = \text{maj. semi-axis of spheroid} \]

\[ \delta W < 0 \iff \alpha > \alpha_{\text{LDM}} \quad \text{stiff} \]
\[ \delta W > 0 \iff \alpha < \alpha_{\text{LDM}} \quad \text{soft} \]

Shells versus excitation

Both shell and anti-shell effects vanish at higher nuclear temperatures.

\[ \delta W \rightarrow 0 \quad \text{for} \quad T \rightarrow \]

Stiff shell nuclei become softer
Soft anti-shell nuclei become stiffer

\[ \text{Jensen-Damgaard 1973} \]
Scission Point Model (SPM)

SPM is intuitively simple but inspiring for discussing energies of FFs.

Scission configuration = two aligned spheroids

\[ V = V_{\text{Coul}} + V_{\text{Def}} = \frac{Z_1 Z_2}{D_1 + D_2 + d} + \alpha_1 (D_1 - R_{01})^2 + \alpha_2 (D_2 - R_{02})^2 \]

Energy set free in fission

\[ Q = \text{TKE} + \text{TXE} = (V_{\text{Coul}} + E_{\text{Kpre}}) + (V_{\text{Def}} + E_{\text{int}}^*) \]

Quasi-static configuration for \( V \) at minimum:

\[ \frac{\partial V}{\partial D_1} = 0 \quad \text{and} \quad \frac{\partial V}{\partial D_2} = 0 \]

\[ \rightarrow \text{calculate for } V \text{ at minimum} \]

\[ \frac{V_{\text{Def1}}}{V_{\text{Def2}}} = \frac{\alpha_2}{\alpha_1} \]

Assume scission occurs for deformation with \( V \) at minimum:

stiff FF1 has small \( V_{\text{DEF1}} \)
soft FF2 has large \( V_{\text{DEF2}} \)
Shells and Anti-shells in Kinetic Energies of Fragments

- Dip in TKE for symmetric fission
  - At mass symmetry: 2 FF with 2 x 120 u
    → 2 soft anti-shell FF strongly elongated
  - low $V_{\text{Coul}}$ and dip in TKE

- Mass asymmetry
  - steered by $^{132}$Sn with Z = 50 and N = 82
  - stiff shell with compact scission config.
  - large $V_{\text{Coul}}$ and hence large TKE

- Superasymmetric Fi
  - $A_{\text{LF}} \approx 80$ u and $A_{\text{HF}} \approx 160$ u
  - soft shell and strong anti-shell
  - elongated scission conf.
  - low $V_{\text{Coul}}$ and low TKE

- Increasing excitation:
  - Shells become softer: TKE
  - Anti-shells stiffer: TKE

Increasing excitation:
Shells become softer: TKE
Anti-shells stiffer: TKE

Straede 1987
Ruben 1991
Shells and Anti-shells in Neutron Emission from Fragments

- Sawtooth of neutron multiplicity $\nu(A)$
  Combine stiff shell at $A_H = 132$ u
  with soft anti-shell at $A_L = 120$ u.
  From SPM:
  $$\frac{V_{\text{Def}}_H}{V_{\text{Def}}_L} = \frac{\alpha_L}{\alpha_H} < 1$$
  Following relaxation of $V_{\text{DEF}}$:
  $$\frac{E^*_H}{E^*_L} < 1$$
  Neutrons drain $E^*$ → $\nu_H / \nu_L < 1$

- For excitation energy increased:
  Stiff shell nuclei become softer and
  Soft anti-shell nuclei become stiffer.
  → $\nu(A)$ sawtooth fades away
  At high $E^*$ n-multiplicity approaches
  smooth increase of $\nu / E^*$ predicted
  by LDM
Y(A) and TKE(A_H) in actinides has 2 components:

**Symmetric and Asymmetric Fission**

Turkevich-Niday: two “modes”
symm. ↔ asymm. evolve independently with excitation

Symm. fission: anti-shells
Asymm. fission: shells

Glendenin 1980

Independent evolution directly observed in TKE(A_H).
Separate Gaussians for symmetric and asymmetric fission.
TKE(A_H) distribution in overlap is broad and skewed.

Holubarsch 1971

**Theory**

PES has 2 distinct paths bifurcating in 2\textsuperscript{nd} minimum.
Down to scission the 2 paths are separated by a high ridge.

Möller 2001

Double-humped barrier on the way from grd. state to scission.
At 2\textsuperscript{nd} saddle the 2 barriers differ in shape and height.
Brosa modes

Bimodal Asymmetric Fission

Fine structure in asymmetric fission:

**Mass Distribution**

![Mass Yield vs Pre-Neutron Mass](image)

- Brosa: Standard I ↔ Standard II
- \( A_H \): spherical shell ↔ deformed shell in Heavy Fragment

**TKE Distribution**

![TKE Distribution](image)

Mode Analysis of \( Y(A) \) and \( \text{TKE}(A) \)

- **SL**: 118, 157(1), 72, anti-shell
- **St I**: 134, 187(1), 82, spherical shell
- **St II**: 142, 167(1), 88, deformed shell

Knitter 1987

Wagemans 1989
Angular Distributions of Fragments

in near barrier (n,f) reactions with (e,e) targets

- Fission prone nucleus near saddle = spheroid
- good quantum numbers are J, M and K
- FF are ejected along axis of elongation: fission axis
- Angular distribution of FF ≡ orientation of fission axis
  \[ W^{J_{MK}}(\theta) \sim |D^{J_{MK}}|^2 \]
  with \( \theta = \angle(n,FF) \) and \( D^{J_{MK}} \) = wavefn of symmetric top

For \( K = \frac{1}{2} \) FF are ejected
  along n beam axis (\( \theta = 0 \))

For \( K = \frac{3}{2} \) and larger FF are ejected
  sideways (\( \theta > 0 \))

\( K \) quantum numbers characterize \( W(\theta) \)
Angular Distributions of Fragments

ABOVE BARRIER

$^{234}\text{U}(n,f)$: fi cross section with $B_f \approx 1.2$ MeV and $B_n = 5.3$ MeV

(n,f) for (e,e) target:
- Target spin $I = 0$
- Neutron spin $s = \frac{1}{2}$
- Neutron orbital $l \perp n$-beam
- Total spin $J = I + l + s$
- Projection on beam $M = \pm \frac{1}{2}$

Turkevich-Niday Modes
Symmetric and asymmetric fission have different barriers. Hence $(J,K)$ numbers differ for symmetric $\leftrightarrow$ asymmetric fission.

$W(\theta)$ symm. fission $\neq W(\theta)$ asymm. fission

where $\theta = \angle (n,\text{FF})$

Brosa Modes
Study for asymmetric fission the anisotropy $W(0^\circ) / W(90^\circ)$ for ST I and ST II

No correlation of angular anisotropy on mass asymmetry is observed.

Indication from A. Bohr postulate:

ST I and ST II are fed by the same transition state
Angular Distributions of Fragments

**SUB BARRIER**

**W(θ) in sub-barrier resonances**

\(^{234}\text{U}(n,f): \) \( E_n = 0.55 \) and \( E_n = 0.77 \) MeV

Characterize \( Y(A) \) by \( \langle A_H \rangle \) and compare \( \langle A_H \rangle \) for \( θ = 0° \) and \( 90° \).

Find \( \langle A_H \rangle_{0°} \neq \langle A_H \rangle_{90°} \)

Example for \( E_n = 0.55 \) MeV resonance:
At \( θ = 90° \) St II/ST I larger than at \( θ = 0° \)

Conclusion:
In tunnel resonance St I and St II are fed at different angles \( θ \) with different \( (J,K) \).

A. Bohr postulate is not valid

**Resonance properties of \(^{234}\text{U}(n,f)\)**

Resonances in \( σ_{fi} \) are due to tunneling being reinforced by \( β \)-vibrations in 2\(^{nd}\) minimum.

Minimum of TKE signals preferential feeding of mode St II (note: \( \langle \text{TKE} \rangle_{St\ II} < \langle \text{TKE} \rangle_{St\ I} \)).

\( E_n = 0.55 \) MeV: St II is reinforced at \( θ = 90° \): \( K = 3/2 \).
\( E_n = 0.77 \) MeV: St II is reinforced at \( θ = 0° \): \( K = 1/2 \).

K of \( β \)-vibrations are \( K = 3/2 \) and \( K = 1/2 \) resp.
**Turkevich-Niday** ↔ **Brosa Modes**

- Symmetric - asymmetric fission ↔ Bimodal asymmetric fission
- Shell ↔ anti-shell ↔ Spherical ↔ deformed shell

- Turkevich-Niday modes differ in transition states → \((J,K)\) differs → \(W(\theta)\) differs

- Brosa ST I and ST II modes in above-barrier fission follow A. Bohr postulate:
  - Identical \((J,K)\) signatures and hence identical \(W(\theta)\) for all masses and TKE in asymm. fission

- For tunnel resonances in sub-barrier fission there is no Bohr-like postulate. Hence
  - St I and St II are fed by different \((J,K)\) leading to different \(W(\theta)\) for the two modes.

- Mass asymmetry and angular anisotropy are correlated

- Corollary: St I and St II bifurcate only once all barriers have been passed
References

[19] A. Turkevich and J.B. Niday, Phys. Rev. 84, 52 (1951)